MATHEMATICS OF YACHT (YET ANOTHER COVID HEALTH TESTING) PROTOCOL FOR EPIDEMIC MANAGEMENT

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The YACHT Protocol provides an evolving tool for imposing structure on the flow of Covid infection information obtained from community testing, collective policy and individual compliance. YACHT Protocol could not assume soundness, invariance, symmetry and completeness of the available information and relied on signaling game theory to design solutions that could evolve with the variable narratives, theories, individual utilities and pathogen variants. Thus, YACHT Protocol suggests a novel and a very flexible pool-testing[5] and badging protocol in the context of controlling contagious epidemics and tackling the far-reaching associated challenges, including understanding and evaluating individual and collective risks of returning prior infected individuals to normal society and other economic and social arrangements and interventions to protect against disease. YACHT Proto uses both control theoretic and game theoretic mathematical models that may be centralized (an optimizing policy maker mandates behavior based on estimated models) or decentralized (a strategizing individual selects their behavior based on available asymmetric information). YACHT protocol demonstrates how society can continue to carry out plausible economic activities in addition to controlling the prevalence of a contagious disease by keeping the number of infected people below a desired limit without compromising an individuals' privacy despite the presence of deception and selfishness among people, and limitations of available resources. Different types of badges would come with different restrictions. Badges would be reissued periodically by third-party testing centers via suitably frequent pool testing of samples of the participants. The size of the pools, frequency of tests, and allowable activities for people with a given type of badge would depend on the available resources, the prevalence of the disease, and the efficacy of the equipment used in the tests.

1. Introduction. Ever since Sir Ronald Ross, a nobel laureate, proposed a mathematical model involving "compartments" of populations – such as S (Susceptible), I (Infected) and R (Recovered), epidemiologist have studied the evolution of epidemics and policies for their containment using ordinary differential equations. But these SIR-like models pose special challenges to policy makers if there is no efficient and accurate procedure to assign an individual to their correct compartment (an asympotomatic infected individual is treated as non-infectious) or to expect compliance where individuals may strategize to maximize utility by signaling membership in a more locally profitable compartment. A crude solution would be as follows: estimate the TPR (test-positivity-ratio) of a population repeatedly by running a biochemical test (e.g., rtPCR or RAT) at a regular interval on a randomly selected subpopulation; if the estimated TPR is above a threshold (e.g., 5 %) impose a lock-down (quarantine all, including large number of non-infectious individuals), but if the estimated TPR is below the threshold lift the lock-down (mobilize all, including some small number of infectious individuals). A more general scheme using a large number of inexpensive pool-testing

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and privacy-preserving badging system could be expected to perform better. We explore the mathematical structure of these generalizations using control theory and game theory. We differ from traditional pool testing that relies on low prevalence to solve sparse instances of the problem using compressed sensing and offers no intelligent policy once a prevalence threshold is crossed.

In other words, we aim to push forth a new paradigm of thinking and policy adaptation to tackling pandemics. We propose a simple mechanism called the YACHT protocol that involves pool testing large amounts of the population and providing a befitting badge that changes according to each test result. The original protocol design is motivated by the following game-theoretic model. Unlike traditional epidemics like the Flu or Chicken Pox, Covid-19 poses a large threat of undetected spread due to a significant portion of the population being asymptomatic [18]. Thus, one cannot determine the type (Infected/Not-Infected) of an individual directly because the signals (temperature, lack of symptoms) are not unique to a type. It forms an interesting signalling game where an individual has to determine who is safe to interact with.[16] This requires an unique signal from each type bringing forth the idea of a personalised badge. The random pool testing mechanism we have developed and analysed through control theoretic means show how we can control spread and keep prevalence under a threshold. But the real world is anything but ideal with the presence of deception, cheating, non-compliance and faking of results which can be modelled through extensive form games by any competent game theorist. By introducing costly signalling (verifiers, protocols, credible and non credible threats) we can push this system towards a separating equilibrium through which will converge to the ideal control theoretic setup as given by the ODE model. The random pool testing mechanism resembles Ethernet's Aloha protocol [9] and Page Rank random surfer approach [10], especially with small/moderate prevalence, which adds to our faith in its robustness. This mechanism can also be adapted to various fields apart from epidemics that ranges from population health statistics which include obesity control and opiod crises to livestock and soil testing in farmlands.

1.1. *Motivation and Overview.* Our paper proposes a novel and versatile game-theoretic mechanism in the context of the COVID-19 pandemic and it's global far-reaching associated challenges. It demonstrates how a society can continue to carry out plausible economic activities in addition to controlling the prevalence of a contagious disease by keeping the number of infected people below a desired limit without compromising an individual's privacy despite the presence of deception and selfishness among people in the face of information asymmetry, imperfect rationality and limitations of available resources. As an example, consider vaccine hesitancy, insufficient global vaccination, vaccine diplomacy, emerging variants of concern resulting in a loss of progress towards herd immunity. [4]

Consequently, the impact of COVID-19 on medical, social, environmental, economic, political, and public health systems has uncovered vulnerabilities in critical infrastructures and exposed dangerous gaps in crisis planning and management across the world, elevating it to a status of a wicked problem. The unprecedented crisis demands a multilateral, interdisciplinary effort collaborating at a global level to advance and adapt science and technology to restore and preserve the world security.[3]

To this end, we have initiated the first steps toward devising and developing mathematical and computational systems of inference, hypothesis testing, and decision making for understanding and evaluating individual and collective risks of returning prior infected individuals to normal society as well as other economic and social arrangements and interventions to protect against disease. In this social-networking age of globalization heading to rapid interaction spanning across continents, it is almost impossible to contain outbreaks or disincentivize harmful echo chambers[8] that hinder dissemination of harmful signals. Harshly imposed restrictions break down the institutions of privacy and freedom which is not sustainable, overall. Furthermore, to develop and impose a mechanism that can efficiently contain and control multiple aspects of outbreaks, epidemics, or pandemics in a real world scenario, we need to acknowledge that the target population would have varied interests, including malicious and self-serving interests that are not present in a hypothetical setting.

This reality has motivated us to consider an appropriate game theoretic set up. Our game theoretic mechanism inherently adapts a market structure along with a badging system that incentivizes acceptable behaviour without breaking privacy of individuals. Different types of badges come with different restrictions. Badges are reissued periodically by third party testing centers via suitably frequent pool testing of saliva samples of the participants. *Pool testing* is a natural and more flexible generalization of individual testing. The idea is to perform one test on a pooled saliva (or another relevant bodily swab) sample obtained from $k \geq 1$ people. k denotes the pool size, and it can be adaptively determined based on the perceived or known disease prevalence level in a locality, associated costs, and feasibility, etc. Pool testing allows us to screen a large number of people while maintaining privacy about individual persons' actual disease status. Furthermore, it can be quite efficient with a quick turnaround while screening people before giving access to enter facilities, or allowing people to mix with others. Perhaps the biggest advantage of pool testing is the cost-effectiveness of the test procedure without compromising any privacy concerns, particularly when the disease prevalence is low. Although pool testing has many pros, some concerns should be taken into consideration while determining the pool size. If a pooled sample tests positive, that does not mean that every participant of that pool is infected. So, intuitively the optimal pool size would decrease as the disease prevalence goes up. On the other hand, if a pooled sample tests negative, then every participant of that pool must be non-infected unless there is a false negative error. Thus, if pooled tests are used for determining the badges, no external entity will be able to know the persons who are infected. At the same time, an infected person alone will be able to decipher his state with ah high confidence as every pooled tests in which he/she participates would result positive. Thus, the disease state of each person would be completely private till he chooses to share his/her badge with an external entity.

The size of the pools, frequency of tests, and the allowable activities for people with a given type of badge depends on the available resources, prevalence of the disease, and efficacy of the equipment used in the tests. Different participants of the economic activities (e.g. business owners, educational institutions, etc.) choose their rules for engaging with people with different badges. The rules eventual success would depend on the volume of economic activities and their reputations about safety. To motivate people refrain from deceiving and breaking protocols, there would be provisions for both credible and noncredible threats. The goal is to maximize freedom while maintaining privacy and also optimise the economic activities. The framework is flexible enough to be compatible for both developed and developing countries. This versatility would enable policymakers visualize the effect of different pandemic controlling strategies on the health and economy of the society, and thereby help them adopt efficient strategies.

1.2. *Existing models (of badging/pooling systems).* **Existing Pooling systems :** There are multiple pool testing[12–14] systems in place. Multi-step Dorfman Pooling [5, 11] and variants require an iterative process to get the results of the individuals. Compressed sensing [2, 15] and encoding[1] based strategies give results in a single round but are severely limited by constraints based on prevalence of contagion in the populace and sensitivity and specificity of the testing machine or process. Almost all these techniques require an individual to be uniquely identified. For example compressed sensing requires solving the system of equations at the authority side to get the result of an individual. Need for a system where only the user can know of their result.

Existing Badging systems : The simplest of badging systems is giving an individual with a positive test result a Red Badge and correspondingly a Green Badge for a negative test result. These badges require an authority to process the result and accordingly give a badge. We posit that each community is unique and needs an efficient system personalised to their needs, constraints and abilities.

Existing systems : Existing systems consider are very myopic (markovian) and only consider the situation at hand. They fail to consider the big picture where testing and its outcomes result in the change of policy and behaviour of individuals in the respective community. There is an underlying disease dynamic and community interaction which form a complex intertwining between the testing and corresponding policy interventions. In the simplest case an increased early testing in a community may prove to be much more effective than cheap efficient testing done later. In this sense it may be beneficial for a community to plan oon performing a 1000 tests on day 1 as compared to 100 tests everyday for the rest of the month. It may also be that the converse is optimal, which is purely dependent on the community and disease at hand. So there is a need for a full fledged system that considers the entire situation from testing, decision making to intervention implementation. This paper focuses on a novel system which combines all these intricacies.

2. A model for a badging system during COVID-19. To track the Covid-19 pandemic or similar highly contagious diseases (with some infected individuals being asymptomatic carriers), ideally, every individual would be tested frequently regularly, unless there is a complete lockdown of the society. But, there are many impediments and constraints, which can make it impossible or unrealistic to test every individual frequently within short intervals. Lack of availability of sufficiently many testing machines, cost of reliable test procedures, the time necessary for obtaining results for such tests, huge population size and lack of awareness about the tests (particularly in some of the developing countries), etc. are examples of such impediments.

In the specific badging system that we analyze in this paper, there are three kinds of badges which would have three different colors: green, orange, and red. People with green badges would have no restriction on their movements and accesses. People having red badges would have total restriction on their movements and accesses, so they would be quarantined. People with orange badges will have a restricted movements and accesses. The extent of restriction would be parameterized. The time between successive tests for a person depends on the color of the badge. People with red (resp. orange) badges will be tested at a lower frequency compared to the people having orange (resp. red) badges, so that the chances of having false positive test outcome are minimized. When a person having a green (resp. orange) badge tests positive or is part of a pool that tests positive, then he/she will get an orange (resp. red) badge after the test. On the other hand, when a person with a red or orange badge tests negative, then he/she gets a green badge after the test.

Anonymous communication to the user could be done as follows: User encrypts and sends to a central server (CS) a “signed” time-stamped message of the QR code of the saliva-vial and user’s cell phone number; the PCR machine encrypts and sends to the same central server another “signed” time-stamped message of the QR-codes used in the pool; CS decrypts both kinds of messages, verifies consistency (time-stamps and QR codes) and sends the results back to the user's cell phones as respective unforgeable badges- the messages to the CS are destroyed. CS creates a central authority, but can be replaced by a distributed ledger (block-chain) or physical wearable wrist tags.

2.1. *Assumptions.* An individual in a population with an existing viral prevalence (percentage of population that is currently infected) gets tested for a virus using a testing machine.

1. Testing involves a testing machine which produces a result within a desired latency (Δ) and with a known false positive rate ($0 < \mu^+ < 1$) and false negative rate ($0 < \mu^- \ll 1$); the cost of the test is a decreasing monotonic function of Δ, μ^+ and μ^-.
2. The result of the test in conjunction with the attributes of the individual will result in a corresponding badge which can be provided virtually. Thus the result may be provided anonymously.
3. The result is based on the disease state of the individual. The individual may get infected on coming into contact with an infected individual. Over time the individual can recover from the infection. The badge of the individual determines his access and restrictions and thus affects his behaviour and state change.
4. The test involves a pooling process where n individuals pool their samples, but use the same testing machine on the combined sample; thus they reduce a potential latency of value $n\Delta$ to Δ, but will increase their false positive rate with the false negative rate almost unchanged; with pooling, the cost and latency can thus be reduced. We consider that the dilution of the sample due to pooling will not affect the sensitivity and specificity while testing the combined sample.
5. Individual can test repeatedly. If pool sizes and prevalence are small then with small number of attempts he will be able to decipher his true state. The change in badge status is appropriately decided.
6. The privacy of an individual in a pool can be ensured. An infected individual will alone be notified of their result through the badge given through multiple pool tests. An external entity cannot decipher the infected individual through a positive pool.
7. To ensure that pools are not inundated with infected agents resulting in most pools testing positive, a testing restriction can be introduced. A rational individual with a negative result will not choose to test again if the μ^- is small enough. Thus an individual with a negative result can be restricted to test immediately with an exponential back-off after repeated negative tests. This is encoded as a restriction based on the badge given to the user.
8. There are no spatial constraints unless specifically stated otherwise. Implying equal access to pools and homogeneous interactions with other individuals.

Note that most of these assumptions can be easily accounted for as parameters but that increases the sheer complexity of analysis. Thus we observe and analyse simple mechanisms to show robustness and effectiveness.

3. Non strategic badging. Here, we consider the idealized scenario, when everyone follows and fully complies with the badging and pool-testing systems and the associated rules for restrictive activities (in case of an orange badge) and quarantining policy (in case of a red badge) irrespective of their economic or other interest. In particular, we assume that people would not fake their disease status to get a favorable badge or violate the rules involving allowable activities. We also assume that human resources (e.g. law enforcement personnel, paramedics, etc.), medical supplies (e.g. test kits), and other logistic issues would not hinder the smooth running of the pool-testing and badging systems.

Fig 1: **Non strategic idealized compartment model of the badging system.**
Each of the 9 compartments correspond to a disease state of an individual and the badge of the individual. The transitions in black denote the change in the disease state of an individual. The Blue and Yellow transitions correspond to change in the Badge of an individual based on the result of a test being positive or negative respectively.

3.1. *ODE system model .* The above notion can be modelled using a system of ordinary differential equations. To study different aspects of the nonstrategic badging and pool-testing system, three main aspects need to be modeled, namely (i) the structure of the network that captures the physical proximity and interactions among people, (ii) the disease spreading mechanism, and(iii) rules for issuing different badges and associated rubrics. In this article, we consider the complete graph as the physical interaction network, the SIR epidemic model as the disease spreading mechanism, and the following badging system consisting of three different kinds of badges (red, orange, green) corresponding to different stipulated freedom and restrictions. Suppose we have a population of size N. The disease status of each person at any time is either S (susceptible), or I (infected), or R (removed). Also, each person carries one of the three possible badges: R (red), or G (green), or O (orange). Combining these two features,it is easy to see that each person has exactly one of the 9 possible states

$$X := \{D_B : D \in \{S, I, R\} \text{ and } B \in \{G, O, R\}\}$$

$$\text{where } \sum D_B = N$$

at every time point. Here, D denotes the disease status of a person, and B denotes the color of the badge that a person carries.

Now we describe how do the states of people evolve. See Figure 1 for a schematic depiction of all the transitions. If a person with state S_G (resp. S_O or S_R) tests positive (resp. negative), then he/she moves to S_O (resp. S_G). Also, S_G-persons become I_G when they get infected. These are the only transitions that control the number of S_G states, so

$$\begin{aligned}\dot{S}_G = &-(\text{rate at which } S_G \text{ tests positive}) + (\text{rate at which } S_O \text{ tests negative})\\ &+ (\text{rate at which } S_R \text{ tests negative}) - (\text{rate at which } S_G \text{ gets infected}).\end{aligned} \tag{1}$$

Next note that if a person with state S_O tests positive (resp. negative), then he/she moves to S_R (resp. S_G). Also, S_O (resp. S_G) persons become I_O (resp. S_O) when they get infected (resp. test positive). These are the only transitions that control the number of S_O states, so

$$\begin{aligned}\dot{S}_O = &-(\text{rate at which } S_O \text{ tests positive}) - (\text{rate at which } S_O \text{ tests negative})\\ &+ (\text{rate at which } S_G \text{ tests negative}) - (\text{rate at which } S_O \text{ gets infected}).\end{aligned} \tag{2}$$

A similar argument gives

$$\begin{aligned}\dot{S}_R = &-(\text{rate at which } S_R \text{ tests negative}) + (\text{rate at which } S_O \text{ tests positive})\\ &- (\text{rate at which } S_R \text{ gets infected}).\end{aligned} \tag{3}$$

Using analogous argument, and taking all possible transitions into consideration (see Figure **??**),

$$\dot{I}_G = -(\text{rate at which } I_G \text{ tests positive}) + (\text{rate at which } I_O \text{ tests negative}) + (\text{rate at which } I_R \text{ tests negative}) + (\text{rate at which } S_G \text{ gets infected}) - (\text{rate at which } I_G \text{ gets removed}), \tag{4}$$

$$\dot{I}_O = -(\text{rate at which } I_O \text{ tests positive}) - (\text{rate at which } I_O \text{ tests negative}) + (\text{rate at which } I_G \text{ tests positive}) + (\text{rate at which } S_O \text{ gets infected}) - (\text{rate at which } I_O \text{ gets removed}), \tag{5}$$

$$\dot{I}_R = -(\text{rate at which } I_R \text{ tests negative}) + (\text{rate at which } I_O \text{ tests positive}) + (\text{rate at which } S_R \text{ gets infected}) - (\text{rate at which } I_R \text{ gets removed}), \tag{6}$$

$$\dot{R}_G = -(\text{rate at which } R_G \text{ tests positive}) + (\text{rate at which } R_O \text{ tests negative}) + (\text{rate at which } R_R \text{ tests negative}) + (\text{rate at which } I_G \text{ gets removed}), \tag{7}$$

$$\dot{R}_O = -(\text{rate at which } R_O \text{ tests positive}) - (\text{rate at which } R_O \text{ tests negative}) + (\text{rate at which } R_G \text{ tests positive}) + (\text{rate at which } I_O \text{ gets removed}), \tag{8}$$

$$\dot{R}_R = -(\text{rate at which } R_R \text{ tests negative}) + (\text{rate at which } R_O \text{ tests positive}) + (\text{rate at which } I_R \text{ gets removed}). \tag{9}$$

The rates appearing in (1)-(9) depend on the parameters of the underlying SIR epidemic model, quarantine and testing policies, etc. We divide the parameters into three groups:

- *Parameters for the freedom of movement policy.* A person with badge B would have freedom level $\phi_B \in [0, 1]$. The freedom levels would be ordered. Green badge holders will have maximum freedom, followed by orange and red badge holders respectively. So, $1 \geq \phi_G \geq \phi_O \geq \phi_R \geq 0$. If a person has freedom level 1, then he/she has full freedom to participate in all economic, social and other activities requiring physical interactions with others. On the other hand, if a person has freedom level 0, then he/she would have no freedom to participate in such activities, and would be quarantined. Intermediate values of freedom level correspond to different extent of restrictions imposed on the allowable activities and movements.
- *Parameters for the testing policy.* Each person needs to participate in the pool-tests, which would be conducted in one of the n pooling stations, periodically on a regular basis to renew their badges. If a person doesn't participate in the pool-test on a day, his/her badge would decay (from green to orange, or from orange to red) automatically. Persons having badge B would be allowed to participate in the pool-tests at rate t_B. These rates are also ordered: $t_G \geq t_O \geq t_R$. The purpose behind this ordering is to reduce false positive outcomes of the pool-tests. Let r be the rate at which pool-tests can be done at each pooling station, and let $f \cdot N$ denote the pool size for each pool-test. So, f denotes the ratio of pool size and population size. The rate at which the population gets tested can be computed in two ways. The total rate of tests is $r \cdot n \cdot f \cdot N$, and the rate at which people arrive at the testing centers is $\sum_{D,B} t_B D_B$. So we must have

$$\sum_{D \in \{S,I,R\}, B \in \{G,O,R\}} t_B D_B = r \cdot n \cdot f \cdot N.$$

- *Parameters for the SIR epidemic model.* Let β (resp. γ) denote the rate of disease transmission (resp. recovery) for the SIR epidemic model.

Based on the above parameters and using $\tau(A, A')$ to denote the transition rate from state $A \in \mathcal{X}$

to state $A' \in \mathcal{X}$, we can rewrite the equations appearing in (1) - (9) more precisely.

$$\dot{S}_G = -\tau(S_G, S_O) + \tau(S_O, S_G) + \tau(S_R, S_G) - \beta \sum_{B\in\{G,O,R\}} \phi_G \phi_B S_G I_B \tag{10}$$

$$\dot{S}_O = -\tau(S_O, S_R) - \tau(S_O, S_G) + \tau(S_G, S_O) - \beta \sum_{B\in\{G,O,R\}} \phi_O \phi_B S_O I_B \tag{11}$$

$$\dot{S}_R = -\tau(S_R, S_G) + \tau(S_O, S_R) + \tau(S_G, S_O) - \beta \sum_{B\in\{G,O,R\}} \phi_R \phi_B S_R I_B \tag{12}$$

$$\dot{I}_G = -\tau(I_G, I_O) + \tau(I_O, I_G) + \tau(I_R, I_G) + \beta \sum_{B\in\{G,O,R\}} \phi_G \phi_B S_G I_B - \gamma I_G \tag{13}$$

$$\dot{I}_O = -\tau(I_O, I_R) - \tau(I_O, I_G) + \tau(I_G, I_O) + \beta \sum_{B\in\{G,O,R\}} \phi_O \phi_B S_O I_B - \gamma I_O \tag{14}$$

$$\dot{I}_R = -\tau(I_R, I_G) + \tau(I_O, I_R) + \beta \sum_{B\in\{G,O,R\}} \phi_R \phi_B S_R I_B - \gamma I_R \tag{15}$$

$$\dot{R}_G = -\tau(R_G, R_O) + \tau(R_O, R_G) + \tau(R_R, R_G) + \gamma I_G \tag{16}$$

$$\dot{R}_O = -\tau(R_O, R_G) - \tau(R_O, R_R) + \tau(R_G, R_O) + \gamma I_O \tag{17}$$

$$\dot{R}_R = -\tau(R_R, R_G) + \tau(R_O, R_R) + \gamma I_R. \tag{18}$$

To obtain the values of $\tau(A, A')$ for different choices of $A, A' \in \mathcal{X}$, we need to first understand the false positive and false negative test outcomes and their probabilities. If λ denotes the effective prevalence, i.e. the probability that a typical person arriving at a testing center is infected, then

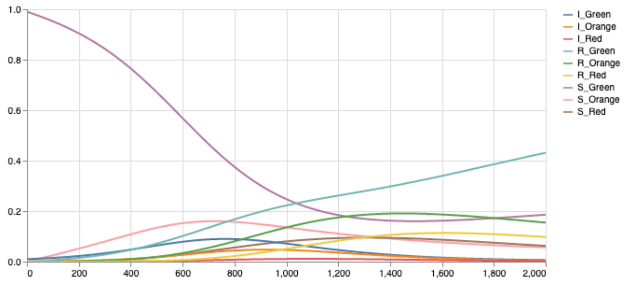
$$\lambda = \lambda(t) = \frac{\sum_{B\in\{G,O,R\}} t_B I_B}{\sum_{B\in\{G,O,R\}, D\in\{S,I,R\}} t_B D_B}.$$

Let μ^+ (resp. μ^-) denote the probability that the result of the testing system is falsely positive (resp. negative) at each time the pool-test is done. Then, from the perspective of a typical person, the effective false positive (resp. negative) probability, i.e. the probability that the result of a pool-test having pool size $f \cdot N$ is falsely positive (resp. negative), will be

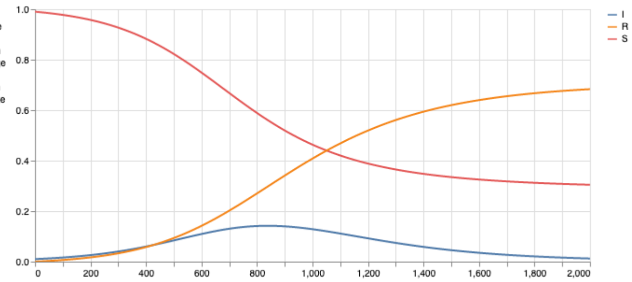
$$\nu^+ := \mu^+(1-\lambda)^{f\cdot N-1} + (1-\mu^-)[1-(1-\lambda)^{f\cdot N-1}] \text{ (resp. } \nu^- := \mu^-).$$

Now, we can obtain the values of $\tau(\cdot, \cdot)$ in terms of the above parameters.

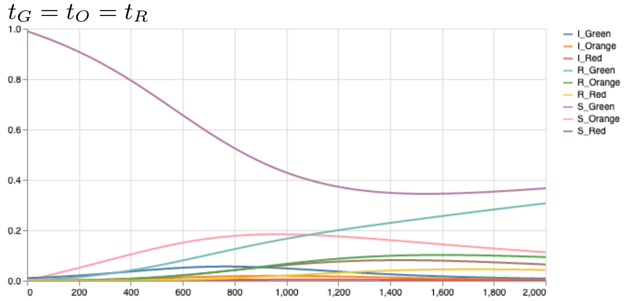
$$\begin{aligned} \tau(S_G, S_O) = \nu^+ t_G S_G, \quad \tau(S_O, S_R) = \nu^+ t_O S_O, \\ \tau(S_O, S_G) = (1-\nu^+) t_O S_O, \quad \tau(S_R, S_G) = (1-\nu^+) t_R S_R, \\ \tau(I_G, I_O) = (1-\nu^-) t_G S_G, \quad \tau(I_O, I_R) = (1-\nu^-) t_O S_O, \\ \tau(I_O, I_G) = \nu^- t_O S_O, \quad \tau(I_R, I_G) = \nu^- t_R S_R, \\ \tau(R_G, R_O) = \nu^+ t_G R_G, \quad \tau(R_O, R_R) = \nu^+ t_O R_O, \\ \tau(R_O, R_G) = (1-\nu^+) t_O R_O, \quad \tau(R_R, R_G) = (1-\nu^+) t_R R_R. \end{aligned}$$



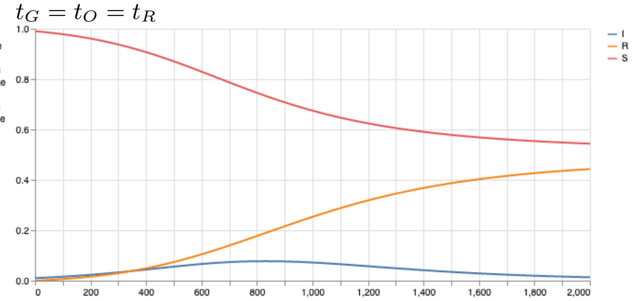
(a) Time series of the 9 compartments when badges are not distinguished for restrictions. $\phi_G = \phi_O = \phi_R = 1$ and $t_G = t_O = t_R$



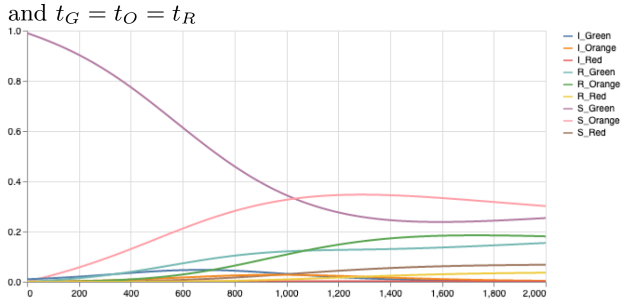
(b) Time series of underlying SIR plot when badges are not distinguished for restrictions. $\phi_G = \phi_O = \phi_R = 1$ and $t_G = t_O = t_R$



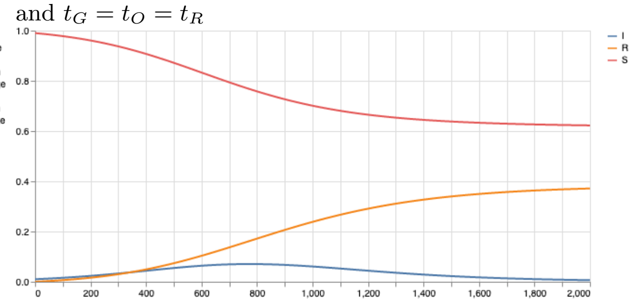
(c) Time series of the 9 compartments when badges are distinguished only for freedom. $1 = \phi_G > \phi_O > \phi_R = 0$ and $t_G = t_O = t_R$



(d) Time series of underlying SIR plot when badges are distinguished only for freedom. $1 = \phi_G > \phi_O > \phi_R = 0$ and $t_G = t_O = t_R$



(e) Time series of the 9 compartments when badges are distinguished for testing and freedom. $1 = \phi_G > \phi_O > \phi_R = 0$ and $t_G > t_O > t_R$



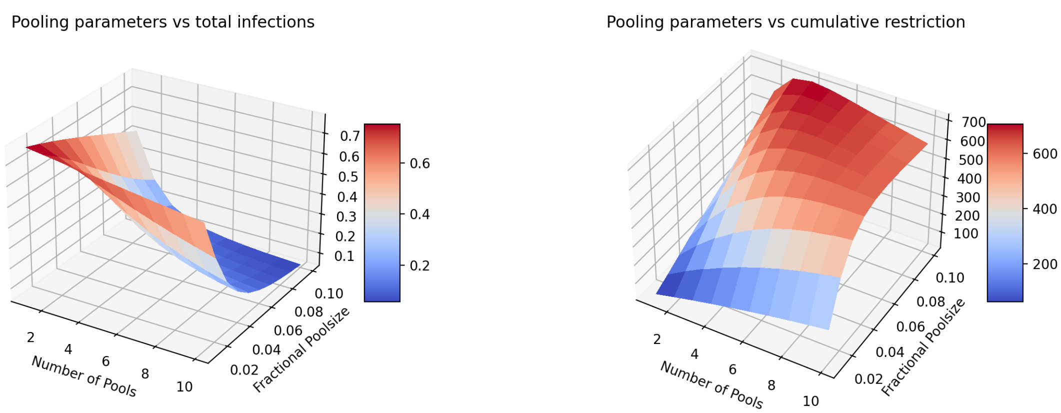
(f) Time series of underlying SIR plot when badges are distinguished for testing and freedom. $1 = \phi_G > \phi_O > \phi_R = 0$ and $t_G > t_O > t_R$

Fig 2: **Comparing the effect of different levels of badging restrictions.**
The subfigures on the left (a),(c),(e) show the timeseries of all 9 compartments while the corresponding subfigures on the right (b),(d),(f) show the underlying SIR plot combining the 9 compartments. From top to bottom we add on badging restrictions gradually to see the effects. In figures (a) and (b), no movement restrictions ($\phi_G = \phi_O = \phi_R = 1$) were imposed implying that all people have equal freedom to move around. In figures (c) and (d), we introduce movement restrictions ($1 = \phi_G > \phi_O > \phi_R = 0$) for people with Orange and Red badges. In figures (e) and (f), we introduce testing restrictions ($t_G > t_O > t_R$) in addition to movement restrictions as in case of figures (c) and (d), so people with Green, Orange, and Red badges are tested at rates which ate descending in order. The disease model considers $\beta = 0.1$ and $\gamma = 0.05$ with initial prevalence of 1%. We have deployed a simple webapp for this model(`https://abcdefg-protocol.herokuapp.com/`).

3.2. *Numerical analysis of ODE system.* Comparing Figure 2(a) (resp. 2(b)) with Figure 2(c) (resp. 2(d)), we see that there is a clear reduction in disease prevalence if movements are restricted for people with orange and red badges, even though the testing rates are the same for all people. Similarly, comparing Figure 2(c) (resp. 2db)) with Figure 2(e) (resp. 2(f)), we see that if restrictions are imposed on testing rates, then there would be a significant decrease in the disease prevalence. Thus, by controlling the freedom of movement of the people and the rates of testing, one can control the disease spread and reduce the disease prevalence. The prevalence can be further reduced by further restricting the freedom of movement. A more fine grained control can be achieved by controlling the pooling strategy, which involves poolsize and number of pools. But this may have some other consequences.

When the parameters associated with the pooling strategy are varied, Figure 3 showcases the trade-off between the total number of infected peoples and the corresponding cumulative restriction enforced on the population. We observe the non-monotonic nature of both total infections and cumulative quarantine. On the one hand, relaxed restrictions on the movement of people would lead to an increase in the total infections, which would lead to higher fatality rates and health risks. On the other hand, stringent restrictions on the movement of people would lead to economic losses both for the people and the governments. This brings forth a question of how to balance between the above two competing costs. This has been formalised in a later section.

Nevertheless, there is a clear indication that the testing and badging systems can keep the disease prevalence under control by manoeuvring the parameters associated with pooling strategy (the pool size and the number of pools), restrictions on the movement of people (ϕ_G, ϕ_O, ϕ_R), and testing rates (t_G, t_O, t_R).



(a) Total infections as a proportion of the population based on the pool testing strategy

(b) Total cumulative restriction among all compartments based on the pool testing strategy

Fig 3: Trade-off between the total number of infected people and the amount of restrictions for a given pooling strategy which is determined by a combination of the number of pools and the sizes of the pools. The x-axis is the number of pools, the y-axis is the proportion of the common size of all the pools and the population size. The z-axis corresponds to the total infections (resp. cumulative restriction) in figure (a) (resp. (b)). The disease model considers $\beta = 0.1$ and $\gamma = 0.05$ with initial prevalence of 1%.

3.3. *Alternate formulation.* Although the ODE system formulation discussed above is relatively easier to understand, and it helps to analyze the long term behavior of the badging system, it cannot answer all important questions regarding the efficiency of the badging system and the underpinning of the parameters involved in the model. It is quite natural to ask whether actually infected (resp. healthy) people would get a red (resp. green) badge quickly or not. How would the average waiting time to get a correct badge depend on the pool size, disease prevalence, etc.? These questions would involve either a discretized analysis[7] of the associated agent based model, or the analysis of the associated Markov chain and the corresponding hitting times.

An alternate way to formulate the badging system on the complete graph is the following continuous-time Markov chain (CTMC) $\{\mathbf{A}(t)\}_{t\geq 0}$, where $A_{BD}(t)$ denotes the number of people having disease status $D \in \{S, I, R\}$ and badge $B \in \{G, O, R\}$ at time t, having the state space

$$\mathcal{S} := \{\mathbf{A} \in \{0, 1, \dots, N\}^{3\times 3} : \sum_{B\in\{G,O,R\}} \sum_{D\in\{S,I,R\}} A_{BD} = N\}.$$

The holding time for the state $\mathbf{A} \in \mathcal{S}$ is exponentially distributed with rate $\sum_{B\in\{G,O,R\}} \sum_{D\in\{S,I,R\}} t_B A_{BD}$. To define the underlying discrete time Markov chain, let $\mathcal{M}$ denote the following set of 18 3×3 matrices indexed by the set of arrows appearing in Figure 1. For each arrow, both the tail and the head belong to $\{G, O, R\} \times \{S, I, R\}$. The matrix associated with the arrow having its tail (resp. head) at (B, D) (resp. (B', D')) has -1 (resp, 1) at the (B, D) (resp. (B', D')) coordinate, and all other entries are zeros. Based on these questions, the set of allowable transitions is

$$\{\mathbf{A} \to \mathbf{B} : \mathbf{A} \in \mathcal{S}, \mathbf{B} = \mathbf{A} + \mathbf{M} \text{ for some } \mathbf{M} \in \mathcal{M}\}.$$

The transition probability $p(\mathbf{A}, \mathbf{A} + \mathbf{M})$ is given by

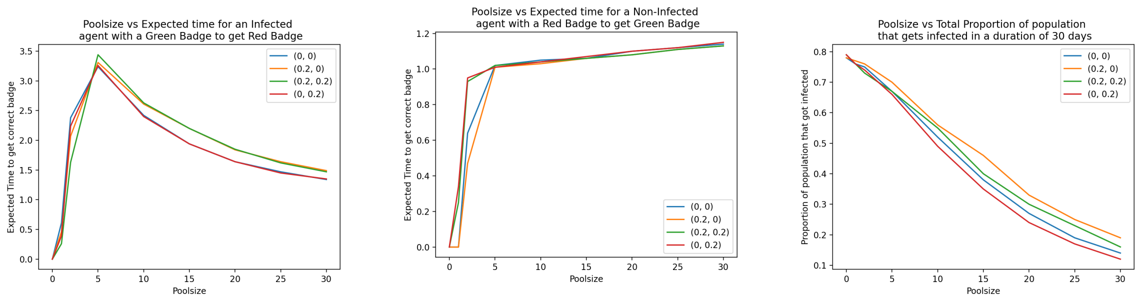
$$p(\mathbf{A}, \mathbf{A} + \mathbf{M}) = \frac{r(\mathbf{A}, \mathbf{A} + \mathbf{M})}{\sum_{\mathbf{M}'\in\mathcal{M}} r(\mathbf{A}, \mathbf{A} + \mathbf{M}')},$$

where $r(\mathbf{A}, \mathbf{A} + \mathbf{M})$ denotes the transition rate (as described in the ODE system) from state $\mathbf{A}$ to state $\mathbf{A} + \mathbf{M}$.

3.4. *Agent based Numerical Analysis .* We have studied how to control the disease prevalence in §3.2, but we haven't studied yet the average time needed by an individual with a wrong badge to get the correct badge. In this section, we carry out a suitable agent based analysis of the complex testing and badging system to obtain realistic estimates of the expected times to get the correct badge from a wrong badge. Figure 4 shows that the expected time needed for a person to get a correct badge, i.e. the expected time needed for an infected (resp. non-infected) person with a green (resp. red) badge to get a red (resp. green) badge, is not monotone in pool size, and it stabilizes if the pool size is increased sufficiently. We see that the total number of infected people is a decreasing function of pool size, as one would expect. We also see that this system is very robust against inherent false positive and false negative testing rates. Even up to a 20% false rate, we see that the total infections can be kept under control without compromising on the expected time to get a correct badge. Also, notice that the expected time to get a green badge for a non-infected person stabilizes fairly quickly and is not perturbed much even if the pool size if increased a lot. Figure 5 further shows that a person would be able to get a correct badge fairly quickly even if the initial disease prevalence is high. Of course this expected time would depend on the initial prevalence. If the initial disease prevalence is high, then, by choosing the pool size appropriately, the total number of infected people can be kept under control without compromising on the time for people to get their correct badges.

3.5. *Cost function.* To answer the question of finding the optimal testing strategy, we define a cost function. There are mainly four kinds of costs that need to be considered:

1. C_1 : cost for quarantining or restricting people – due to a Red or Orange badge agents will be restricted and thus will not be able to take part in economic activities involving physical interactions. This cost encodes the economic loss.



(a) Average hitting time for Infected to get Red if originally Green

(b) Average hitting time for Non-Infected to get Green if originally Red

(c) Corresponding plot of proportion of population that gets infected

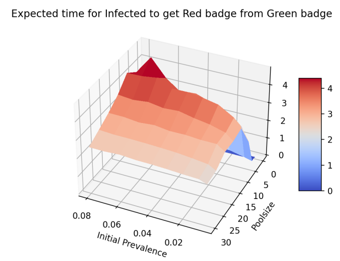
Fig 4: This figure depicts the simulation results about the expected waiting time to get the correct badge based on an "agent-based" version of the testing and badging model. Due to the stochastic nature of the agent-based models, the epidemic process has been simulated multiple times. The outcomes are averaged over multiple instances to reduce the variance and focus on the mean behavior. We analyse the expected times to get a correct badge as the pool size is varied. The x- axis for all subfigures correspond to the pool size. The y-axis for the subfigure (a) (resp. (b), (c)) correspond to the expected time to get a correct green badge (resp. expected time to get a correct red badge, the proportion of people who get infected within 30 days). Different color corresponds to different combinations of false positive and false negative testing rates, The robustness and effectiveness of this mechanism can be observed even up to an inherent 20% false positive or false negative rate in the testing machine. This agent based simulation consists of 1000 agents with disease parameters $\beta = 0.0003$ and $\gamma = 0.08$ over a duration of 30 days.

2. C_2 : cost incurred by the infected people – this cost stems from the fact that the infected people with health cost dues to hospitalizations, health hazards, and fatalities. This cost encodes the medical and logistical costs for an infected person.
3. C_3 : cost for a testing pool – this is a two part cost. One cost stems from setting up the testing machines which conduct tests. The second part consists of the logistic and reagent costs for operating the pool.
4. C_4 : fixed costs – this cost encodes the fixed costs that include setting up the badging system and other processes like information dissemination.

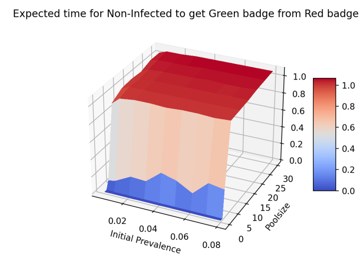
The cost function is thus a linear sum of the cost components $C_1 + C_2 + C_3$. Note that for optimization, we do not have to consider C_4 as it is a constant fixed cost.

Goal : Find the optimal pool and badge parameters that minimize the cost function.

3.6. *Optimality.* The total cost associated with a pandemic within a certain time period depends on many factors. The disease prevalence, fatality rates, economic costs because of restrictions on the movement of people which in turn depend on the pooling strategy, additional constraints, etc. share a very convoluted nonlinear relationship with the total cost. The constraints and priorities vary across different communities. Different communities may have different sets of constraints, and they can set parameters according to their needs, constraints, preferences, or abilities. For example, in case of an old age home or nursing home, the fatality risks are quite high. So, in that case a lower disease prevalence rate would be desirable, as the total cost would be very sensitive to the disease prevalence. On the other hand, in case of a community of youngsters, for whom the fatality rates are



(a)



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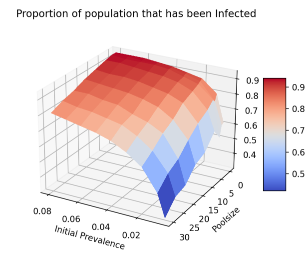

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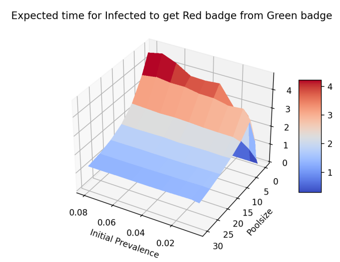
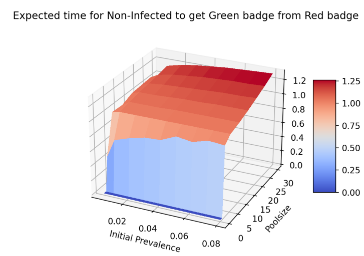
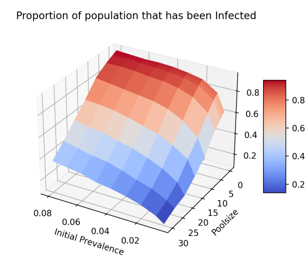


(d)

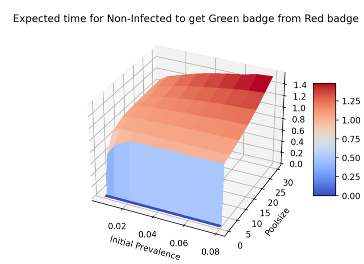


(e)

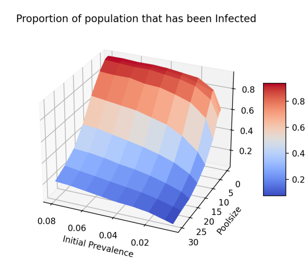


(f)

(g)



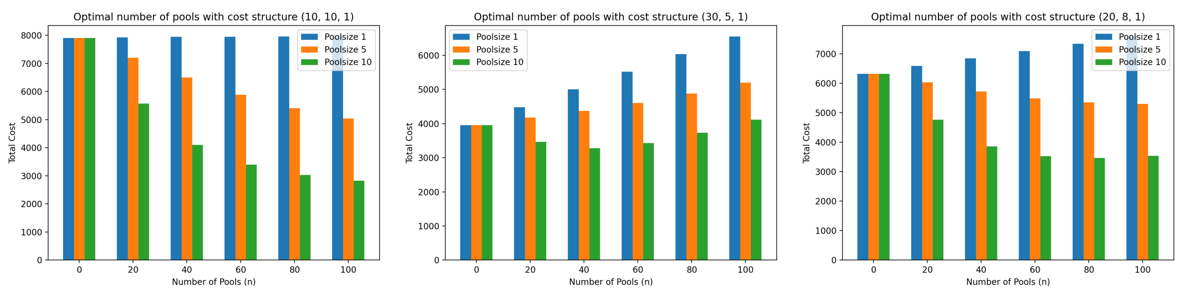
(h)



(i)

Fig 5: This Figure depicts the following surface plots associated with the agent-based simulation conducted in §3.4. For all subfigures, the x-axis corresponds to the initial prevalence, and the y-axis corresponds to the pool size. For the top (resp. middle, bottom) row of subfigures, the number of pools is 10 (resp. 30, 50). The z-axis for the left column of subfigures (i.e., subfigures (a), (d), (g)) denotes the expected time for an infected person to get a red badge from a green badge. The z-axis for the middle column of subfigures (i.e., subfigures (b), (e), (h)) denotes the expected time for an non-infected person to get a green badge from a red badge. The z-axis for the right column of subfigures (i.e., subfigures (c), (f), (i)) denotes the proportion of population that gets infected over the duration of the simulation (30 days). This agent based simulation consists of 1000 agents with disease parameters $\beta = 0.0003$ and $\gamma = 0.08$ over a duration of 30 days.

relatively low and who need to get out for their education or profession, or in case of people, who can earn only when they are out in the public places, it would be better to relax movement restrictions to reduce the economic losses rather than trying to reduce disease prevalence only. Constraints can arise from availability of tests, vaccines, availability of funds, or even medical equipment. The abilities of a community is the technical and logistical power available at hand. In Figure 6, we consider a highly infectious epidemic that may not be controlled with single agent testing using the provided testing machines. This results in a peculiar situation, where doing the tests would not reduce the overall cost, if one can only perform normal tests with one person at a time. We observe that, in case of our badging based pooling system, there would be a drastic decrease in cost to the society despite an inherent 10% false positive and 10% false negative testing rates. Note that when the pool size is 1, i.e. when each person is tested separately, then the person who tests positive would immediately have a red badge and is restricted accordingly. Thus we see that our mechanism is significantly better than individual testing with pool size 1. Furthermore, we observe that the cost functions are not necessarily monotonic and thus the optimal number of pools and optimal pool size can occur (need not occur) at the boundary values.



(a) Total cost given pooling strategy for cost 10 per testing machine, 10 for an infected individual and 1 per unit of quarantine.

(b) Total cost given pooling strategy for cost 30 per testing machine, 5 for an infected individual and 1 per unit of quarantine.

(c) Total cost given pooling strategy for cost 20 per testing machine, 8 for an infected individual and 1 per unit of quarantine.

Fig 6: This figure demonstrates how would the total cost depend on different pooling strategies and cost structures. The three subfigures correspond to different cost structures. Different color corresponds to different pool size. For all subfigures, the x-axis (resp. y-axis) corresponds to the number of pools (resp. total cost). The optimal number of pools depend on the cost structure and the pool size. We see that 100 (resp. 40, 80) pools with pool size 10 is the optimal number of pools for the cost structure considered in (a) (resp. (b), (c)). The disease model considers $\beta = 0.1$ and $\gamma = 0.05$ with initial prevalence of 1%.

4. Strategic Badging. Strategic Badging is realistic agent level modelling where an agent chooses a decision that maximises one's payoff. Unlike the previous section the costs borne by the agent depends on their respective utility. The goal is to translate this individual utility to a community utility.

Strategic decision making leads to deception with certain malicious agents. Furthermore the general notion of compliance is completely reliant on the utility of the population. To hinder such deceptive strategies and incentivize compliance requires authorities to enforce penalties. this can be carried out through verifiers who check the status of individual. Furthermore, individual may have the ability to contest the results of a test or consequently the given badge by retesting by an idealized (oracle) machine with negligible $\Delta \approx 0$ and $\mu^{+,-} = 0$; however such a machine when used (e.g, rarely) will be VERY costly.

Our goal is simple. Through costly signalling, to nudge the system into a good separating equilibrium where the types (infected, non-infected) are easily distinguishable. In that case we converge to the ideal system that behaves as the above ODE model.

Fig 7: Non strategic idealized from Figure 1 to show the transitions corresponding to faking a badge.

4.1. *ODE with faking badge.* Int he real world, a person can choose to fake their badge. Rationally agents will only fake a Red badge for a Green badge. We can model this as a δ proportion of population choosing to fake their badges. This can be achieved by modifying 3 existing transitions of our ODE model resulting in 6 of the original ODE equations being changed as follows. We represent the new rates of change for our compartments with

$$\dot{D}'_B, \text{ where } D \in \{S, I, R\}, B \in \{G, O, R\}$$

Equations of the compartments representing Green badges (10),(13),(16) will all be modified as follows.

$$\dot{D}'_G = \dot{D}_G + \delta D_G \text{ , where } D \in \{S, I, R\}$$

Equations of the compartments representing Red badges (12),(15),(18) will all be modified as follows.

$$\dot{D}'_R = \dot{D}_R - \delta D_R \text{ , where } D \in \{S, I, R\}$$

This captures the deceptive process at an aggregated population level but cannot capture the individual level decision making. For this we look towards a game theoretic approach using an extensive form representation as shown in the following section.

4.2. *Badging as an Extensive form game.* We start with a simple game and work our way up. Figure 8, encodes a simple game where an agent on testing in a pool is given a Red or Green Badge.

Fig 8: A one shot game, where an agent on testing in a pool gets a Red badge or a Green badge. Note that the costs here are that incurred by the agent and not the community as captured by the cost function in the previous section. The Nash Equilibrium of this game varies based on the associated costs. Our goal is to ensure an equilibrium which minimises our cost function that encodes a tradeoff between infections and restrictions.

Ideally we require the Red Badge to quarantine but the agent with a Red Badge makes the decision by maximising their payoff. Thus the agent will choose to quarantine only when the expected payoff is higher than the expected payoff from interaction.

$$-C_t - C_q > \frac{(1-\lambda)\nu^+(-C_t) + \lambda(1-\nu^-)(-C_t - C_i)}{(1-\lambda)\nu^+ + \lambda(1-\nu^-)}$$

Thus to incentivize Red Badges to quarantine, badges can be checked before entry to interaction spaces. But this results in agents choosing to fake their badges. To counteract this we introduce a verifier or checker who checks the badges of those interacting. This is analogous to a policemen randomly checking the license of drivers or a more stringent bouncer checking the ID's of entrants. The check will be done with a near oracle testing machine with false positive and false negative rates close to zero and on testing positive an agent is penalized. Not that such a change also affects the payoff of infected agents who on getting a false negative had gotten a Green badge. This changes the above inequality as follows.

$$-C_t - C_q > \frac{(1-\lambda)\nu^+(-C_t) + \lambda(1-\nu^-)(-C_t - C_i - C_p(P(\text{getting checked})))}{(1-\lambda)\nu^+ + \lambda(1-\nu^-)}$$

Here C_p is the penalty borne by agent after testing positive when checked by the verifier. The probability of getting checked depends on the frequency of verification which is a cost borne by the community or authority. This brings forth another game where the authority optimises between the cost of enforcing quarantine for Red badges through verification and the payoff of reduced disease spread. This game is not trivial as the population utility is not homogeneous and thus shall be discussed in a follow up paper.

The extensive form representation cannot capture the dynamic nature of this mechanism. Each action of every player(agent, verifier, authority) involved results in a state change of the environment. As the badges expire requiring repeated testing, we encounter a repeated game with a non stationary prevalence dependent cumulatively on all historical actions.

5. Discussion.

5.1. *Translation of results to application.* One has to account for inherent heterogeneity when looking at a real world population. For example heterogeneous susceptibility of agents based on blood group(Blood group A more susceptible to Covid while O is least susceptible[6, 17]). Thus parameters like immuno-genetic profile, vaccination status, demography has to be embedded into the personalised badge of the individual. We have shown an adaptive testing cum restriction strategy brought out through badging. But real world implementation calls for a more practical details and an easy and feasible method to be implemented. A more practical variant is currently being tried out by HealthBadge, Inc. with pilot projects in college campuses in India, thus, showing the real world feasibility of such a system.

5.2. *Extensions to badging parameters.* One of the novel ideas of our paper is the introduction of a suitably designed, highly customizable, and efficient badging system. The badging system is a method that would facilitate the task of decision making for the policy makers. Such a system would be instrumental to determine how to impose restrictions on movement of people without violating individuals' privacy concerns or damaging the economy significantly.

Every person would be assigned a virtual and dynamic badge based on the person's latest test result and other features which are informative for tackling pandemics. For instance, the badging system can account for immuno-genetic profiles of the people, demographic information and medical history including vaccination. The badge of a person can also depend on the badges of those who would come in close contact with him/her frequently. The badges would ensure the privacy of individuals, and at the same time restrict the infected (and potentially infected) people from spreading infection effectively. The badging system would be highly customizable and can incorporate requirements specific to enforcing institutions (e.g., university or college dorms, educational institute campus) based on the institution-specific information.

Furthermore, if information from some contact tracing apps or individuals' movement is available, the badging system can be modified to make it compatible with such tracking data so that high-risk individuals can be given restrictive badges, the badges of people coming in contact with other high-risk individuals can be degraded, etc. Thus people in contact with high-risk people would get restrictive badges by either increasing their quarantining periods or penalization in some other way. Such a badging system can be adjusted to innately punish those who violate guidelines such as meeting other people despite having a restrictive badge or going to high-risk locations.

The badging system can also be very helpful for running economic and other activities. Entry regulations can be set up for locations like shopping malls, restaurants, educational institutes, religious places, entertainment venues, etc. The regulation policies can be decentralized so that individual location authority can strategize their own rules for allowing people for accessing their facilities based on their priorities, reputation, and cost-benefit analysis.Examples of restrictions that can be imposed using the badging system include(a) a maximum of 10 orange badges at any time, (b) separate entrance or restricted area for people having different badges, (c) allowing only green badges, etc.

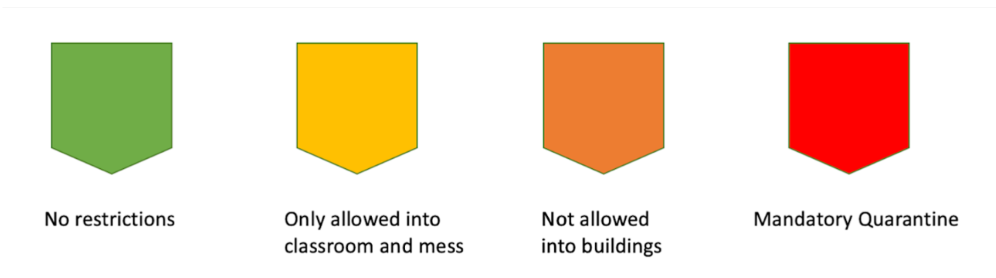


Fig 9: Example badging system for an university campus

Thus, the badging system can become instrumental to balance between controlling risks and running the economic and other necessary activities by enforcing minimal restrictions. Essentially creating an efficient method to enforce restrictions without violating privacy. Due to pooling, no external entity will be able to determine the agent who is positive. At the same time a positive agent alone will be able to decipher his state with ah high confidence as every pool he tests in will turn up positive. Thus an agent's state is completely private till he chooses to share his badge with an external entity. This can be done to gain access to restricted locations that only allow agents who are not infected. Furthermore, policy makers can make economic decisions and can adjust badging rules accordingly. For example, a policy can increase the number of badges to accurately cover a wide range of people. Furthermore the decay rate can be increased to make people test more often. Thus it is a single point mechanism that allows for a complete picture when it comes to tackling an epidemic.

5.3. *Change point analysis.* A variant of the testing and badging system that we consider in this paper can be effective in detecting sudden changes in the disease prevalence that may occur because of some super spreading events or appearance of new strains with higher infectivity and/or enhanced health risk. Here, we have considered a closed system where the total number of people is fixed. But, the analysis can be generalized to an open system, where people are allowed to join and leave a community. If there is a sudden surge in the number of infected people from outside, or if the parameters associated with the epidemic model change, then there would be a significant change in the number of orange and red badges issued by the pooling stations.This information can be used to detect changes in the disease prevalence, and then appropriate measures can be taken (e.g. restricting movements of people) temporarily to bring the disease prevalence down. To realize the consequences of different measures, the testing and badging system can be analyzed starting from different initial disease prevalence.

5.4. *Independent mechanism with Market Structure.* In this paper, we have considered the case where the pool size is the same across pools. There can be many generalizations which are more realistic. In general, the size of the pools can be set independently of other pools. Each pool may have its own false positive and false negative rates, which can be different from those of other pools. Note that the pool size can be dynamically changed as well. If the disease prevalence is very low (resp. high), then one can have a high (resp. low) pool size without compromising on the false positive rate. Another important generalization of our badging and testing system is the case, when the pools have spatial locations. People may not visit randomly chosen pools, but may be interested/asked to visit pools closest to them. The pooling stations can be run by non-governmental agencies, and each pool would receive a reputation based on the cost charged and parameters which would determine how many people would choose to come to the pooling station.

5.5. *Restaurant Game.* If the testing and badging system is implemented, then shops, restaurants, commercial entities can use the badge information of people, who wants access to their places, and devise their own policies regarding who would be allowed in their facilities. For example, one can choose to allow only green badges, or one can choose to allow only up to 10 orange badges at a time, etc. To determine the best policy, each commercial entity needs to consider a game theoretic set up and obtain the strategy that would optimize the profit. On the one hand, if the policy for allowing people is too stringent and restrictive, then there would be too few customers and the business would suffer heavy economic losses. On the other hand, if the policy is too relaxed, then the place can potentially becomes an infection hotspot and the reputation of the commercial entity would be at stake. If it can be tracked or believed that infection spreads from a shop, then there would be a loss of reputation for that shop which will affect future customers. That may compel the authorities to shut down the commercial activities, or customers themselves may avoid the facility altogether. Along with these concerns, the commercial entities would be involved in a competitive zero sum game which should be taken into consideration to determine their policies, because if one entity gains a customer, then others lose that customer.

6. Acknowledgements. We thank RxCovea (https://rxcovea.org/) a worldwide multidisciplinary research for useful comments and encouragements. Additionally. we acknowledge HealthBadge, Inc. (https://www.healthbadge.org/) for it's MVPs and pilot projects that show the real world feasibility and practical application of this system. We acknowledge Vijay Chandru, Charles Cantor and Jantine Broek for helpful discussions and key insights.

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